A metaheuristic schema for the Inventory Routing Problem

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1 Introduction

The Inventory Routing Problem (IRP) consists in defining vehicle routing operations and inventory management over a time horizon, considering a set of customers C with known deterministic demands per time period and maximum and minimum limits of inventory level, a set of homogeneous vehicles V with a limited capacity Q, and a time horizon P with discrete time periods. The IRP aims to define the quantities to be delivered to each customer, the periods to do so, and the order to visit the customers using the available vehicle fleet so that the demands of customers are satisfied at a minimum inventory management and transportation cost. In addition, the capacity of vehicles and the inventory level limits of customers must be respected.

The classical IRP is proposed by [1], and many variations of the problem are presented [2]. The originality of this work consists in solving the IRP using a GRASP-ELS metaheuristic approach with a Dynamic Programming (DP) algorithm embedded. Feasible solutions are generated by a two-step method. The first step defines a giant tour encompassing the number of products to deliver to all the customers over the entire time horizon. In the second step, a DP algorithm called Split determines the routes made by the vehicle fleet at each time period. The proposed method generates good-quality solutions in a reasonably low running time, even for large-scale instances.

This work is organized as follows. Section 2 describes the two-step method which generated feasible solutions. Section 3 presents the GRASP-ELS metaheuristic approach, encompassing the two-step method. Lastly, conclusions and intended future works are presented.

2 The two-step heuristic

The two-step heuristic proposes feasible solutions with a controlled rate of randomness, aiming to diversify the initial solutions for the GRASP-ELS. In the first step, a giant tour is generated with a defined sequence of customers and delivery quantities. Then, the DP algorithm evaluates this sequence and provides an optimal solution for the IRP given the giant tour. The next section presents both methods respectively.

2.1. Giant tour generation

The giant tour T is defined on a directed graph $G = (V', A)$, in which V' corresponds to the vertex set, where $V' = V \cup \{0\}$, and A the arc set. Each node $v \in V$ is represented by a triplet $\{c_v, q_v^p, p\}$, in which c_v corresponds to the customer of node v, q_v^p to a predefined quantity of products to be delivered until the time period p. The delivery quantities over the time horizon are defined based on the initial inventory level and demands of customers so that they are made in the latest possible time period and without a stockout. For example, given a customer $i \in C$ with an initial inventory level of s_i^0 and demands d_i^p, $\forall p \in P'$, where P' is a subset of P. The first delivery of customer i will be made at period $p = |P'| + 1$, for the time horizon P' in which inequality (1) is not verified. Afterward, the sequence in which customers are visited on the giant tour is defined respecting the chronological order of deliveries. Customers that are delivered at the same time period are randomly ordered in the giant tour.

$$s_i^0 - \sum_{p \in P'} d_i^p \geq 0 \qquad (1)$$

2.2. Dynamic programming approach

Given a giant tour, a feasible solution for the IRP is generated by a DP approach that can be seen as a shortest path with resource consumption. The method consists in splitting the giant tour T into routes from the end to the beginning by adding feasible arcs between pairs of nodes (i, j), where $i, j \in T$ and $i < j$. This procedure allows the anticipation of deliveries in case the time period of i is less than that of j. The algorithm contains two key ideas:

- the optimal splitting of T into routes per time period, and
- the anticipation of the quantities to be delivered.

Figure 1 shows an example of the giant tour before and after evaluation, respectively.

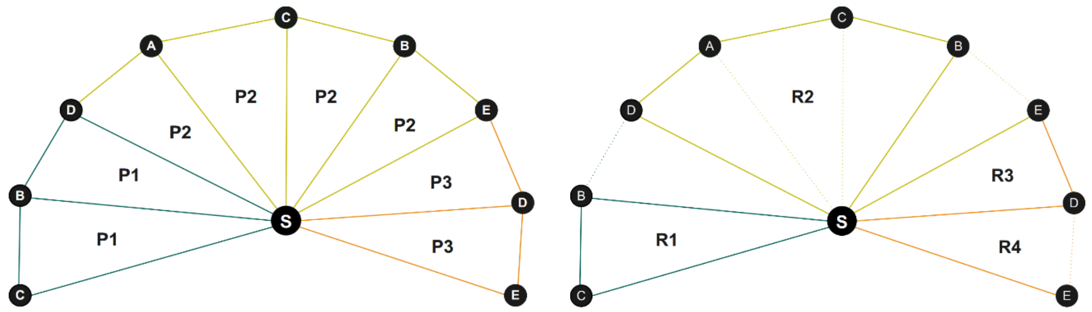

Figure 1 - Giant tour before and after evaluation

We can observe from route R1 that the delivery to customer A is previously defined for period 2 in the giant tour. However, through the DP algorithm, it is anticipated to the first period providing low transportation costs. The same is valid for route R3 in which the delivery to customer D is previously defined for period 3, but is anticipated for period 2 in the IRP solution. The period of each route is defined by the period of the first node in the route. This algorithm can obtain the optimal splitting given a giant tour. The DP approach is based on the one proposed by [3] for the Vehicle Routing Problem (VRP) with a heterogeneous fleet of vehicles.

3 The GRASP-ELS metaheuristic

The Greedy Adaptative Search Procedure (GRASP) and Evolutionary Local Search (ELS) metaheuristic proposed is illustrated in Figure 2. It consists of generating an initial giant tour, then its evaluation using the DP algorithm and applying local search operators presented below. Then, at the ELS levels, successive neighbor solutions are created. For each neighbor solution, a perturbation consists of permutating some customers into the giant tour is applied, followed by the evaluation procedure and the local search operators. At the end of each ELS level, the best solution is computed, and a new iteration is started from this best solution obtained so far.

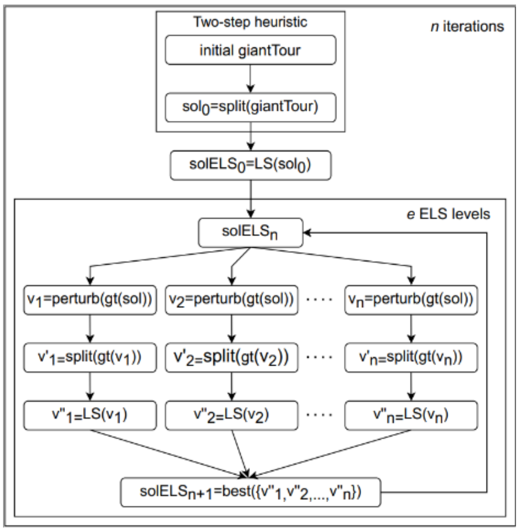

Figure 2 - GRASP-ELS

3.1. Local search operators

In order to improve the quality of solutions obtained from the two-step method presented previously, some local search operators have been selected to be applied based on empirical tests. The operators are presented below.

- 2-OPT intra-route: in a given route, two non-adjacent arcs are removed, and two more are added to form a new route.
- 2-OPT inter-routes: two arcs of different routes performed at the same period are removed, and two more are added to form two new routes.
- Reinsertion intra-route: one customer is removed and reinserted in another position of the same route.
- Reinsertion inter-routes: one customer is removed from a route and reinserted in a position of a different route performed at the same time period.

The local search procedure considers at the beginning equal probabilities for all operators. Throughout the method execution, the probabilities are progressively updated depending on the success of each operator in generating better solutions. These probabilities are restarted at each execution of the local search procedure.

The cost of a solution obtained by using intra-route or inter-routes operators for routes made at the same time period is quickly calculated. Otherwise, for moves of customers between routes performed at different time periods, an extra verification must be done to recalculate inventory levels and verify if the inventory capacity of customers and the production capacity of the supplier are still being respected.

3.2. Preliminary results

Initial computational experiments were conducted for a set of benchmark instances proposed by [1] with up to 30 or 50 customers within 3 or 6 time periods. The tests were conducted on a computer equipped with a 3.10 GHz Intel Core i5-10500 processor and 16 GB of RAM. The methods were implemented in C++ language, and 10 executions were performed for each instance considering 30 ELS levels, 5 neighbor solutions generated at each ELS level and at most 3 permutations.

Table 1 summarizes the obtained results. Column V corresponds to the number of vehicles considered, C to the number of customers, $ttt(s)$ the time to obtain the best feasible solution, and column $tt(s)$ the total time of the metaheuristic execution. We obtain an average gap of around 5%, even for large instances.

V	C	$ttt(s)$	$tt(s)$
2	5	0,74	16,07
2	10	3,08	16,72
2	15	2,62	17,67
2	20	12,81	21,21
2	25	7,59	21,43
2	30	12,73	23,99
2	35	18,11	28,53
2	40	24,90	33,23
2	45	25,58	31,48
2	50	38,91	43,51
			Av. gap = 5.50%
3	5	0,649	16,2678
3	10	6,3284	16,704
3	15	9,2386	17,4544
			Av. gap = 4.02%

Table 1 - Results

4 Conclusion and perspectives

In this work, we proposed a DP approach embedded in GRASP-ELS metaheuristic to solve the IRP. To our knowledge, the proposed method is the first split algorithm that considers a multi-period giant tour with inventory constraints for the IRP. The GRASP-ELS metaheuristic provides average gaps of up to 5.5% in a very low execution time for instances available in the literature.

We intend to develop a metaheuristic based on a DP approach to solve a new benchmark of instances, including other characteristics such as heterogeneous fleets of vehicles and inventory costs that vary per time period. These characteristics are intrinsic to real systems. Otherwise, they increase the complexity of the problem. New local search operators must be tested to ensure the quality of the obtained solutions, including those that consider multi-period aspects.

References

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